(English Version)

Instructions : *1. The question paper has five Parts A, B, C, D and E. Answer **all** the **Parts**.*

2. Use the Graph Sheet for the question on Linear Programming in Part-E.

PART – A

Answer **all** the **ten** questions : **($10 \times 1 = 10$)**

1) Let $*$ be the binary operation on N, given by $a * b = LCM$ of a and b. Find $20 * 16$.

2) Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$.

3) Construct a 2×2 matrix, $A = [a_{ij}]$, where elements are given by, $a_{ij} = \frac{i}{j}$.

4) If A is a square matrix with $|A| = 8$ then find the value of $|AA'|$.

5) If $y = \cos\sqrt{x}$, find $\frac{dy}{dx}$.

6) Find $\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$.

7) Define collinear vectors.

8) Find the direction cosines of a line which makes equal angles with the positive co-ordinate axes.

9) Define feasible region in a linear programming problem.

10) If A and B are independent events, $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$ then find $P(A\cap B)$.

PART – B

Answer **any ten** questions : **(10 × 2 = 20)**

11) If $f:R\to R$ defined by $f(x)=1+x^2$, then show that f is neither 1-1 nor onto.

12) Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right)=2\cos^{-1}x$, $\frac{1}{\sqrt{2}}\le x\le 1$.

13) Solve the equation $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$, $(x>0)$.

14) Find the values of k, if area of triangle is 4 sq. units and vertices are $(k, 0)$, $(4, 0)$ and $(0, 2)$ using determinant.

15) If $ax+by^2=\cos y$, find $\frac{dy}{dx}$.

16) Verify Rolle's theorem for the function $f(x)=x^2+2x-8$, $x\in[-4,2]$.

17) Find the approximate change in the volume of a cube of side x metres caused by increasing the side by 3%.

18) Integrate $\frac{\tan^4\sqrt{x}\sec^2\sqrt{x}}{\sqrt{x}}$ with respect to x.

19) Evaluate $\int_0^{2/3} \frac{dx}{4+9x^2}$.

20) Find the order and degree of the differential equation

$\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - \sin^2 y = 0$.

21) Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively, in the ratio 2 : 1

i) Internally.

ii) Externally.

22) Find the area of the parallelogram whose adjacent sides are determined by the vector $\vec{a} = \hat{i}-\hat{j}+3\hat{k}$ and $\vec{b} = 2\hat{i}-7\hat{j}+\hat{k}$.

23) Find the vector and Cartesian equation of the line that passes through the points $(3, -2, -5)$ and $(3, -2, 6)$.

24) Find the probability distribution of number of heads in two tosses of a coin.

PART – C

Answer **any ten** questions : **(10 × 3 = 30)**

25) Show that the relation R in $\mathbb{R}$ (set of real numbers) is defined as $R = \{(a,b): a \leq b\}$ is reflexive and transitive but not symmetric.

26) Write $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \neq 0$ in the simplest form.

27) If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if $AB = BA$.

28) Differentiate $(\log x)^{\cos x}$ with respect to x.

29) Differentiate $\sin^2 x$ with respect to $e^{\cos x}$.

30) Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

31) Evaluate : $\int \frac{2x}{x^2 + 3x + 2} dx$.

32) Evaluate : $\int e^x \sin x \, dx$.

33) Find area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$.

34) Form the differential equation of the family of circles having centre on y-axis and radius 3 units.

35) Find x, such that the four points $A(3, 2, 1)$, $B(4, x, 5)$, $C(4, 2, -2)$ and $D(6, 5, -1)$ are coplanar.

36) Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, evaluate $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $\left|\vec{a}\right| = 1$, $\left|\vec{b}\right| = 4$ and $\left|\vec{c}\right| = 2$.

37) Find the shortest distance between the lines $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$.

38) Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

PART – D

Answer **any six** questions : **(6 × 5 = 30)**

39) Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where S is the range of f, is invertible. Find the inverse of f.

40) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ prove that $A^3 - 6A^2 + 7A + 2I = 0$.

41) Solve the following system of linear equations by matrix method.

$$x - y + 2z = 1$$
$$2y - 3z = 1$$
and $3x - 2y + 4z = 2$.

42) If $y = \left(\tan^{-1} x\right)^2$. Show that $\left(x^2 + 1\right)^2 y_2 + 2x\left(x^2 + 1\right)y_1 = 2$.

43) The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When $x = 8$ cm and $y = 6$ cm, find the rate of change of

i) The perimeter and

ii) The area of the rectangle.

44) Find the integral of $\sqrt{x^2 - a^2}$ with respect to x and hence evaluate $\int \sqrt{x^2 - 8x + 7}\, dx$.

45) Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

46) Solve the differential equation $\cos^2 x \frac{dy}{dx} + y = \tan x \left(0 \le x < \frac{\pi}{2}\right)$.

47) Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.

48) The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

i) none

ii) not more than one

iii) more than one

will fuse after 150 days of use.

PART – E

Answer **any one** question : **(1 × 10 = 10)**

49) a) Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x)dx$. **(6)**

b) Show that :

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx).$$ **(4)**

50) a) Minimize and Maximize $z = 600x + 400y$

Subject to the constraints :

$x + 2y \leq 12$

$2x + y \leq 12$

$4x + 5y \geq 20$ and $x \geq 0, y \geq 0$ by graphical method. **(6)**

b) Find the value of k, if

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & x \neq \dfrac{\pi}{2} \\ 3, & x = \dfrac{\pi}{2} \end{cases}$$

is continuous at $x = \dfrac{\pi}{2}$. **(4)**